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JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA ~~CALIFORNIA~~

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EFFICIENCY OF FISSION ELECTRIC CELLS

Clifford J. Heindl

Frank B. Estabrook

Frank B. Estabrook, *Chief*
Physics Section

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA
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ABSTRACT

Electrical efficiencies are calculated for fission-electric cells of parallel plate, concentric sphere, and concentric cylinder geometries as a function of operating voltage and fuel layer thickness. For spherical and cylindrical cells, several ratios of outer to inner radii are included, covering the range which appears feasible for a reactor which is to be carried on a spacecraft. The calculations are simplified by ignoring the distribution of fission fragment masses, charges, and kinetic energies and utilizing average values for these quantities; a linear rate of energy loss is assumed for the first portion of the fragment trajectories, as they pass through the fuel layer. The calculated efficiencies decrease with fuel layer thickness, increase with the curvature of the electrodes and ratio of outer to inner electrode radii, and exhibit a maximum at operating voltages near one-half of maximum achievable potential.

I. INTRODUCTION

Work is in progress at a number of laboratories on the problem of producing electricity from fission in a nuclear reactor without using a thermodynamic cycle. One purpose of this work is the development of a lightweight power supply functioning without rotating machinery for use in space applications.

One concept which has been advanced during the course of this development is the direct conversion of the kinetic energy of the charged fission fragments into electrical energy in a fission electric cell (Ref. 1). A simplified version of such a cell is shown in Fig. 1. Two metal plates are separated by a vacuum gap wide enough to prevent voltage breakdown between the plates; on the upper surface of the lower plate is a layer of fissionable material. The cell is within, or may constitute one fuel element of, a nuclear reactor. Neutrons striking the fuel layer on the lower plate cause fission to occur, normally creating two high-energy charged fragments per nucleus split. These fragments have an average energy of about 80 mev each and a positive charge equal in magnitude to approximately 20 electron charges. Half of the fragments will start out in the direction of the upper half plane, the others being lost in the fuel layer backing plate. Since the fragment range is very short, on the order of 13 (mg/cm^2) in uranium, a significant fraction will penetrate the fuel layer and reach the vacuum gap only if the layer is extremely thin. Those fragments which reach and cross the gap will create a positive charge on the upper plate and a net negative charge on the lower—in effect, charging the cell as a condenser. The maximum potential which can be created by the fragments is approximately 80/20 = 4 mv. Steady-state operation at any lower potential can be achieved by connecting to an external load, as shown in Fig. 1, matched to the internal current flow of fission fragments.

One important measure of the practicality of such a device is its electrical efficiency: that is, the fraction of total fission power converted to electrical power. This efficiency must be comparable to that achieved by thermoelectric or thermionic converters used in conjunction with nuclear reactors or the concept will not be competitive for space applications. Calculated efficiencies of fission electric cells are already available in the literature for parallel plate (Ref. 1) and some cylindrical cases (Ref. 2). In the following sections these calculations are partially repeated and extended to further cylindrical and several spherical configurations. The intent is to bring together for easy reference the calculated efficiencies over the ranges of voltage, geometry, and fuel layer thickness which appear applicable to space systems, all obtained under the same simplifying assumptions, utilizing the same method of calculation. The assumptions and method are detailed below, along with the results obtained.

II. ASSUMPTIONS

Several simplifying assumptions have been made to reduce the calculational problem to easily manageable form. These assumptions are:

1. *Uniformity of fission fragments:*

   All fission fragments are assumed to be of a single mass and a single, constant charge and to start out at a single energy with isotropic distribution.

2. *Linear rate of energy loss:*

   It is assumed that the energy of a fission fragment traversing fuel material is $E = E_0(1 - r/\lambda)$, where E_0 is the starting energy, r the distance traveled since birth, and λ the extrapolated range in fuel material, which is approximately one-half the actual range. The linear assumption seriously overestimates rate of energy loss at low energies, but is very good over that portion of the fragment range which is important here.

3. *Effectively plane sources:*

   The curvature of the fuel layer in spherical and cylindrical cells has been ignored in computing the rate and energy of fission fragment emission from the fuel surface. All cases are treated as planar.

4. *Unimportance of edge effects:*

   Both plane and cylindrical cells have been treated as if the elements were infinite in extent.

5. *Steady-state operation:*

   Current was presumed to be flowing through the external circuit at a constant rate equal to the internal flow of charged particles between plates, giving a constant voltage across the cell.

6. *Extraneous charge flow ignored:*

   It is assumed that all electrons ejected from the fuel layer in the slowing-down process are prevented from reaching the anode, and that there is no net flow of Compton electrons in either direction. The current is taken to be equal to the rate of fission fragment traversal multiplied by average fission fragment charge, the counterflow of decay betas being ignored.

For an exact calculation of output from a particular cell, it would be necessary to eliminate most of these assumptions. However, none of them appears to introduce gross errors; calculations on this basis are certainly adequate for comparison of geometric configurations, cell voltages, and fuel layer thicknesses.

III. CALCULATION OF EFFICIENCY

A. Definition of Cell Efficiency

The electrical efficiency of a fission electric cell can be defined as the fraction of total fission power which becomes available as electric power output. For steady-state operation, electric power output per unit area of fuel layer can be calculated as the product of V, the fixed voltage differential between anode and cathode, e', the charge on each fission fragment, and η, the number of fission fragments per second reaching the anode from each square centimeter of fuel surface.

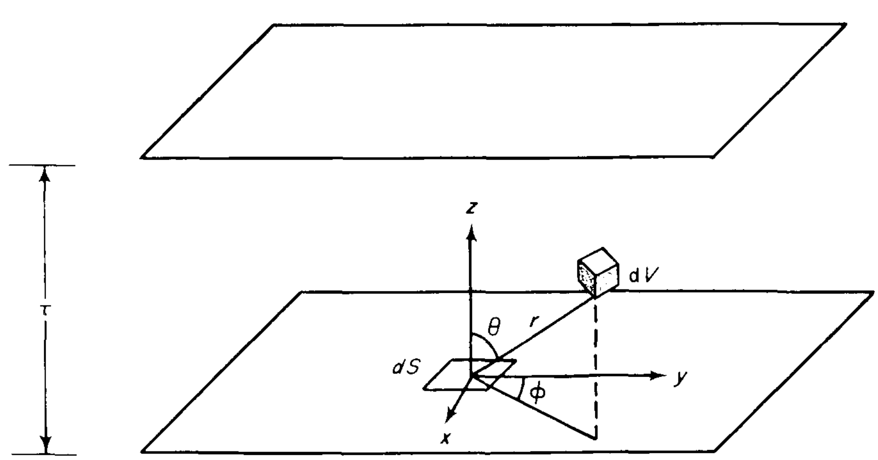
The corresponding total fission power is that produced in one square centimeter of fuel layer—that is, the product of τ, the fuel layer thickness, N, the number of fission fragments produced per cubic centimeter per second in fuel, and E' the fission energy associated with each fission fragment. With two fragments per fission, E' is just one-half of the energy per fission which is actually deposited in the reactor system—that is, total energy per fission minus the energy carried away by escaping neutrinos, neutrons, and gamma rays.

Electrical efficiency $\mathscr{E}$ can thus be written:

$$\mathscr{E} = \frac{Ve'\eta}{\tau NE'}$$

For any given cell configuration and operating condition, ϵ follows immediately from η. However, η is a function of V, τ, and geometry, and must be calculated as shown below.

A portion of the fuel layer can be drawn as follows:



where the lower plane represents the fuel-vacuum interface through which fission fragments pass on their way to the anode. The upper plane is the fuel layer–structural material interface. Fission fragments from the arbitrary volume element dV within the fuel layer traverse the distance r in fuel material and pass through the surface element dS. In spherical coordinates,

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

B. Zero Voltage Case

For the simple case of cell voltage $V = 0$, the number of fission fragments per second passing through the surface and reaching the anode is

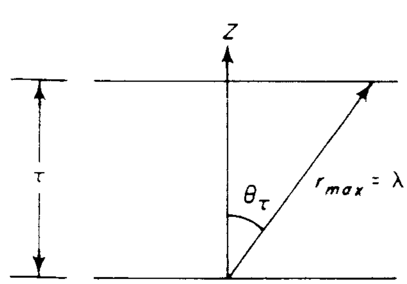
$$d\eta_0 = \frac{N \, dS \cos\theta}{4\pi r^2} \, dV \qquad \text{if} \qquad r \leq \lambda$$

$$d\eta_0 = 0 \qquad \text{if} \qquad r > \lambda$$

If $\tau \geq \lambda$, a very simple result is obtained:

$$\eta_0 = \frac{N}{4\pi} \int_0^{\pi/2} d\theta \int_0^{\lambda} dr \int_0^{2\pi} d\phi \sin\theta \cos\theta = \frac{N\lambda}{4} \; \frac{\text{fission fragments}}{\text{cm}^2 \text{ sec}}$$

the region of integration being merely a hemisphere of radius λ. If $\tau < \lambda$, the rate of flow of fission fragments through the fuel surface will be limited by fuel layer thickness rather than λ over the range $0 \leq \theta \leq \theta_\tau$:



$$\cos\theta_\tau = \frac{\tau}{\lambda}$$

and the region of integration is now a truncated hemisphere:

$$\eta_0 = \frac{N}{4\pi}\left(\int_{\theta_\tau}^{\pi/2} d\theta \int_0^{\lambda} dr \int_0^{2\pi} d\phi \sin\theta\cos\theta + \int_0^{\theta_\tau} d\theta \int_0^{\tau\sec\theta} dr \int_0^{2\pi} d\phi \sin\theta\cos\theta\right)$$

$$= \frac{N\tau}{2}\left(1 - \frac{\tau}{2\lambda}\right)\frac{\text{fission fragments}}{\text{cm}^2\ \text{sec}}$$

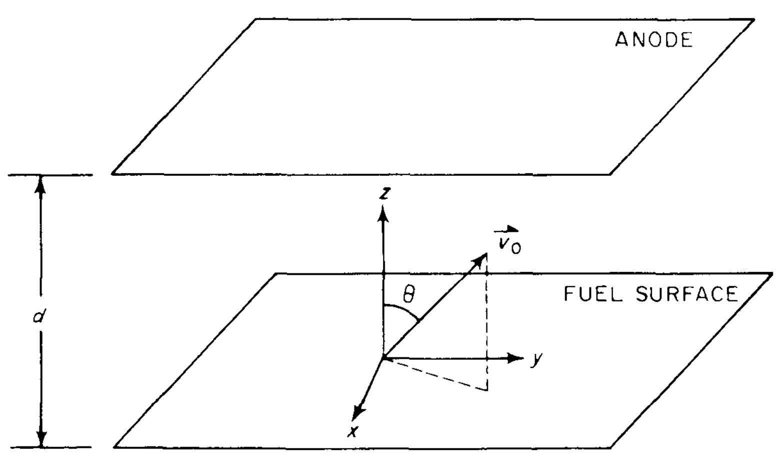
C. Finite Voltage Case

For the situation of actual interest, where a voltage V exists between anode and fuel layer, the electric field created by this voltage hinders the passage of fission fragments across the vacuum gap to the anode, giving a new $\eta < \eta_0$. In order to cross the vacuum gap against this opposing field, a fission fragment must emerge from the fuel layer with energy $E \geq E_{min}$, where E_{min} is a function of voltage, geometry, and angle of emission. According to the linear energy loss assumption, this means that all fission fragments originating in the region $0 \leq r \leq r_{max}$, $r_{max} = \lambda\ [(E_0 - E_{min})/E_0]$, will reach the anode. Those originating in the region $r_{max} \leq r \leq \lambda$ will pass through the fuel-vacuum interface, but will be turned back by the electric field. For $r > \lambda$, the fission fragments will, of course, be stopped in the fuel layer.

The minimum emergent energy necessary to overcome the electric field must be found as a function of V and angle of emergence for each geometry:

1. Parallel Plate Electrodes

Since the potential is a function of z alone, the x and y components of velocity are constant.



Therefore, since energy is conserved:

$$\frac{1}{2} M (v_z)^2_{z=0} = \frac{1}{2} M v_0^2 \cos^2 \theta = \frac{1}{2} M (v_z)^2_{z=d} + Ve'$$

where v_z is the z component of velocity. A minimum energy fragment, just able to reach the anode, will have $(v_z)_{z=d} = 0$, giving the relation:

$$E_{min}(\theta, V) = \frac{1}{2} M (v_0)^2_{min} = \frac{Ve'}{\cos^2 \theta}$$

Combined with the assumption of linear energy loss in penetrating fuel material:

$$E_{min} = E_0 \left(1 - \frac{r_{max}}{\lambda}\right) = \frac{Ve'}{\cos^2 \theta}$$

or

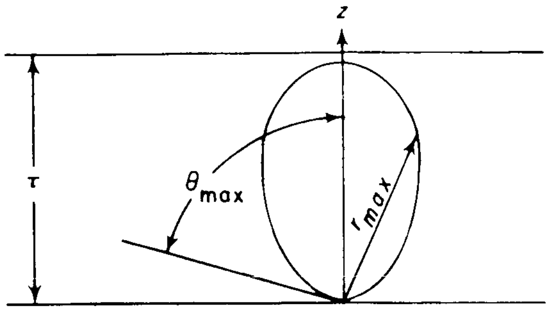
$$r_{max} = \lambda \left(1 - \frac{Ve'}{E_0 \cos^2 \theta}\right)$$

It can be seen that, for the $V > 0$ case, there also exists a maximum angle of useful emission, $\theta_{max} \leq \pi/2$, since even the most energetic fragment will have too small a z component of velocity to penetrate the electric field if it is emitted at any angle greater than

$$\frac{1}{2} M (v_0)^2_{max} \cos^2 \theta_{max} = E_0 \cos^2 \theta_{max} = Ve'$$

$$\theta_{max} = \cos^{-1} \left(\frac{Ve'}{E_0}\right)^{1/2}$$

Thus, for a thick fuel layer, the region of integration is now



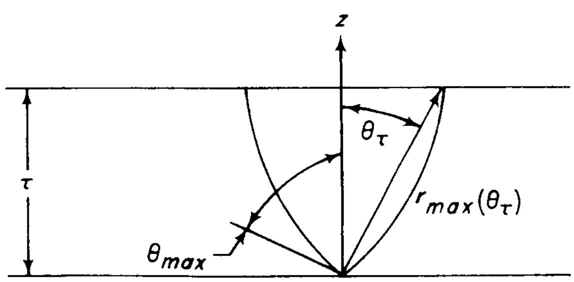
where θ_{max} could also be obtained by setting $r_{max} = 0$. This is physically identical to the argument above, since E_0 is just the energy of a fragment which has penetrated zero thickness of fuel material.

Just as in the $V = 0$ case, the two possibilities of energy-limited, and fuel-layer-thickness-limited fragment currents must be considered. The criterion which divides the thick-layer (energy-limited) from the thin-layer case is now:

$$\tau \geq \lambda \left(1 - \frac{Ve'}{E_0}\right) \qquad \text{thick layer}$$

$$\tau < \lambda \left(1 - \frac{Ve'}{E_0}\right) \qquad \text{thin layer}$$

That is, for the thick-layer case $\tau \geq r_{max}$ for all angles θ, and the fragment current is equal to that which would be obtained for a fuel layer of infinite thickness. For the thin-layer case, there exists an angle θ_τ such that for all $\theta \leq \theta_\tau$ the fragments are capable of penetrating the full thickness of fuel layer and still reaching the anode:



$$\frac{\tau}{\cos\theta_\tau} = r_{max} = \lambda\left(1 - \frac{Ve'}{E_0\cos^2\theta_\tau}\right)$$

or

$$\cos\theta_\tau = \frac{1}{2}\left(\frac{\tau}{\lambda} + \sqrt{\frac{\tau^2}{\lambda^2} + 4\,\frac{Ve'}{E_0}}\right)$$

The fragment current reaching the anode can thus be written:

$$\eta = \frac{N}{4\pi}\int_0^{\theta_{max}} d\theta \int_0^{r_{max}} dr \int_0^{2\pi} d\phi \,\sin\theta\cos\theta \tag{1}$$

for

$$\tau \geq \lambda\left(1 - \frac{Ve'}{E_0}\right)$$

$$\eta = \frac{N}{4\pi}\int_0^{\theta_\tau} d\theta \int_0^{\tau\sec\theta} dr \int_0^{2\pi} d\phi\,\sin\theta\cos\theta + \int_{\theta_\tau}^{\theta_{max}} d\theta \int_0^{r_{max}} dr \int_0^{2\pi} d\phi\,\sin\theta\cos\theta \tag{2}$$

for

$$\tau < \lambda\left(1 - \frac{Ve'}{E_0}\right)$$

where

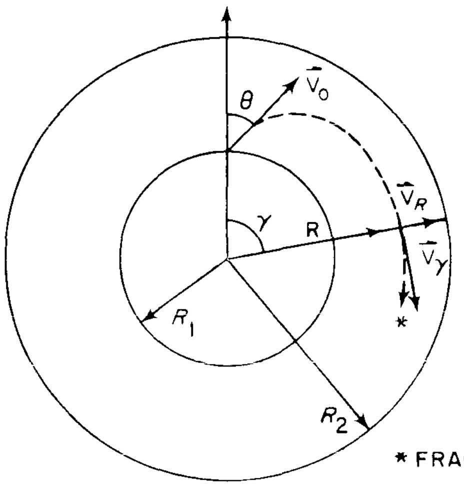
$$\theta_{max} = \cos^{-1}\left(\frac{Ve'}{E_0}\right)^{1/2}$$

$$r_{max} = \lambda\left(1 - \frac{Ve'}{E_0\cos^2\theta}\right)$$

$$\theta_\tau = \cos^{-1}\left(\frac{\tau}{2\lambda} + \frac{1}{2}\sqrt{\frac{\tau^2}{\lambda^2} + 4\,\frac{Ve'}{E_0}}\right)$$

2. Concentric Sphere Electrodes

The fuel layer is assumed to be on the outer surface of the inner sphere, the inner surface of the outer sphere acting as the fission fragment collector. Since the potential supplies a central force field, each fragment trajectory will lie completely in a single azimuthal plane:



Total angular momentum as well as energy is conserved in the central field:

$$\frac{1}{2} M v_0^2 = \frac{1}{2} M (v_R)^2_{R=R_2} + \frac{1}{2} M (v_\gamma)^2_{R=R_2} + Ve'$$

$$M R_1 (v_\gamma)_{R=R_1} = M R_1 v_0 \sin\theta = M R_2 (v_\gamma)_{R=R_2}$$

Combining the above:

$$\frac{1}{2} M v_0^2 = \frac{1}{2} M (v_R)^2_{R=R_2} + \frac{1}{2} M \frac{R_1^2}{R_2^2} v_0^2 \sin^2\theta + Ve'$$

A minimum energy fragment, just able to reach the anode, will have $(v_R)_{R=R_2} = 0$, giving:

$$E_{min}(\theta, V) = \frac{1}{2} M (v_0)^2_{min} = \frac{Ve'}{1 - \frac{R_1^2}{R_2^2} \sin^2\theta}$$

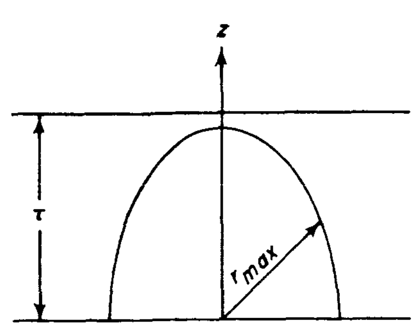
With the assumption of linear energy loss in traversing fuel material, this becomes:

$$E_{min} = E_0 \left(1 - \frac{r_{max}}{\lambda}\right) = \frac{Ve'}{1 - \frac{R_1^2}{R_2^2} \sin^2 \theta}$$

or

$$r_{max} = \lambda \left(1 - \frac{\frac{Ve'}{E_0}}{1 - \frac{R_1^2}{R_2^2} \sin^2 \theta}\right)$$

One expects that there will also exist a θ_{max} beyond which it is energetically impossible for an emitted fragment to reach the anode, just as in the case of plane electrodes. However, the situation in the spherical case is somewhat more complex. Substituting $\sin \theta = 1$ in the above, it is seen that r_{max} remains greater than zero even at $\theta = \pi/2$, provided $(Ve'/E_0) < (1 - R_1^2/R_2^2)$. That is, because of the spherical geometry of the electrodes, it would be energetically possible for a fragment to reach the anode even when initially directed at an angle $\theta > \pi/2$, provided the resisting potential is less than the critical amount above. This is physically impossible, however, since an inward-directed fragment will simply be stopped in the fuel layer; for the assumption of effectively plane sources, all angles $\theta > \pi/2$ are forbidden. Therefore, it is necessary to set $\theta_{max} = \pi/2$ for all V such that $(Ve'/E_0) < (1 - R_1^2/R_2^2)$, and the fragment current is surface-limited. The region of integration is then:



When $(Ve'/E_0 \geq (1 - R_1^2/R_2^2)$, r_{max} goes to zero for some $\theta_{max} \leq \pi/2$, beyond which it is not energetically possible for the fragment to reach the anode against the potential V. In this case, the region of integration is identical to that for plane electrodes. Setting $r_{max} = 0$ gives

$$\sin^2 \theta_{max} = \frac{R_2^2}{R_1^2}\left(1 - \frac{Ve'}{E_0}\right)$$

or

$$\theta_{max} = \sin^{-1}\left(\frac{R_2}{R_1}\sqrt{1 - \frac{Ve'}{E_0}}\right)$$

for this potential-limited case.

Just as with the parallel plate electrodes, there again exist thick-layer (energy-limited) and thin-layer (fuel-thickness-limited) cases determined by:

$$\tau \geq \lambda \left(1 - \frac{Ve'}{E_0}\right) \qquad \text{thick layer}$$

$$\tau < \lambda \left(1 - \frac{Ve'}{E_0}\right) \qquad \text{thin layer}$$

The dividing angle θ_τ is now found from

$$\frac{\tau}{\cos \theta_\tau} = r_{max} = \lambda \left(1 - \frac{\dfrac{Ve'}{E_0}}{1 - \dfrac{R_1^2}{R_2^2}\sin^2 \theta_\tau}\right)$$

which is a cubic equation in $\cos \theta_\tau$ or $\sin \theta_\tau$.

The fragment current reaching the anode for the above four spherical cases is

$$\eta = \frac{N}{4\pi}\int_0^{\pi/2} d\theta \int_0^{r_{max}} dr \int_0^{2\pi} d\phi \sin \theta \cos \theta \tag{3}$$

for

$$\tau \geq \lambda \left(1 - \frac{Ve'}{E_0}\right); \qquad \frac{Ve'}{E_0} < \left(1 - \frac{R_1^2}{R_2^2}\right)$$

$$\eta = \frac{N}{4\pi} \int_0^{\theta_{max}} d\theta \int_0^{r_{max}} dr \int_0^{2\pi} d\phi \sin\theta \cos\theta \tag{4}$$

for

$$\tau \geq \lambda \left(1 - \frac{Ve'}{E_0}\right); \qquad \frac{Ve'}{E_0} \geq \left(1 - \frac{R_1^2}{R_2^2}\right)$$

$$\eta = \frac{N}{4\pi} \left(\int_0^{\theta_\tau} d\theta \int_0^{\tau \sec\theta} dr \int_0^{2\pi} d\phi \sin\theta \cos\theta + \int_{\theta_\tau}^{\pi/2} d\theta \int_0^{r_{max}} dr \int_0^{2\pi} d\phi \sin\theta \cos\theta \right) \tag{5}$$

for

$$\tau < \lambda \left(1 - \frac{Ve'}{E_0}\right); \qquad \frac{Ve'}{E_0} < \left(1 - \frac{R_1^2}{R_2^2}\right)$$

$$\eta = \frac{N}{4\pi} \left(\int_0^{\theta_\tau} d\theta \int_0^{\tau \sec\theta} dr \int_0^{2\pi} d\phi \sin\theta \cos\theta + \int_{\theta_\tau}^{\theta_{max}} d\theta \int_0^{r_{max}} dr \int_0^{2\pi} d\phi \sin\theta \cos\theta \right) \tag{6}$$

for

$$\tau < \lambda \left(1 - \frac{Ve'}{E_0}\right); \qquad \frac{Ve'}{E_0} \geq \left(1 - \frac{R_1^2}{R_2^2}\right)$$

where

$$\theta_{max} = \sin^{-1} \left(\frac{R_2}{R_1} \sqrt{1 - \frac{Ve'}{E_0}} \right)$$

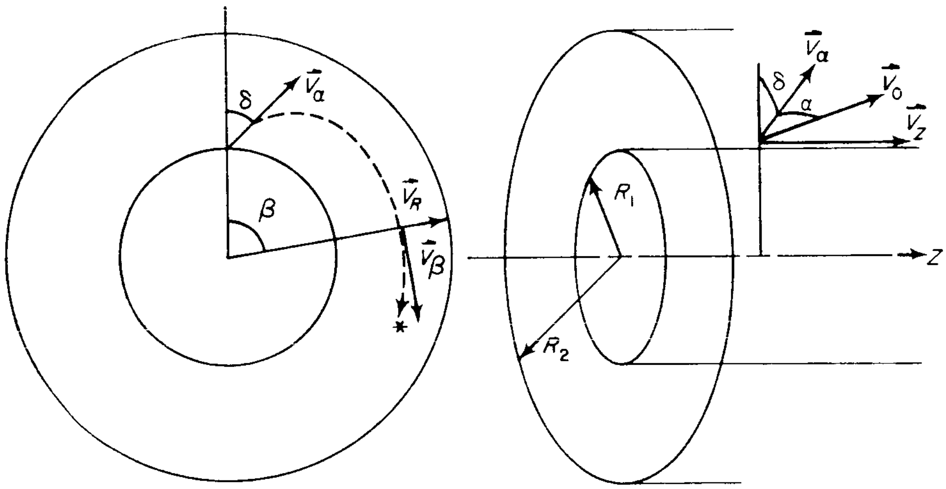
$$r_{max} = \lambda \left(1 - \frac{\dfrac{Ve'}{E_0}}{1 - \dfrac{R_1^2}{R_2^2} \sin^2 \theta} \right)$$

and θ_τ is found from:

$$\frac{\tau}{\cos \theta_\tau} = \lambda \left(1 - \frac{\dfrac{Ve'}{E_0}}{1 - \dfrac{R_1^2}{R_2^2} \sin^2 \theta_\tau} \right)$$

3. Concentric Cylinder Electrodes

The fuel layer is again taken to be on the outer surface of the inner electrode.



* FRAGMENT TRAJECTORY PROJECTED ON $Z = 0$ PLANE

Since the potential is a function of R alone, the z components of momentum and of angular momentum will be conserved, as well as the energy:

$$\frac{1}{2} M v_0^2 = \frac{1}{2} M (v_\beta)^2_{R=R_2} + \frac{1}{2} M (v_R)^2_{R=R_2} + \frac{1}{2} M (v_z)^2_{R=R_2} + Ve'$$

$$(v_z)_{R=R_2} = (v_z)_{R=R_1} = v_0 \sin \alpha$$

$$MR_2 (v_\beta)_{R=R_2} = MR_1 (v_\beta)_{R=R_1} = MR_1 \sqrt{v_0^2 - (v_z)^2_{R=R_1}} \sin \delta$$

Combining the above:

$$\frac{1}{2} M v_0^2 = \frac{1}{2} M (v_R)^2_{R=R_2} + \frac{1}{2} M v_0^2 \sin^2 \alpha + Ve' + \frac{1}{2} M \frac{R_1^2}{R_2^2} v_0^2 \cos^2 \alpha \sin^2 \delta$$

A minimum energy fragment, just able to reach the anode, will have $(v_R)_{R=R_2} = 0$, giving:

$$E_{min} = \frac{1}{2} M (v_0)^2_{min} = \frac{Ve'}{\cos^2 \alpha \left(1 - \frac{R_1^2}{R_2^2} \sin^2 \alpha\right)}$$

With the linear energy loss assumption in passage through fuel material:

$$E_{min} = E_0 \left(1 - \frac{r_{max}}{\lambda}\right) = \frac{Ve'}{\cos^2 \alpha \left(1 - \frac{R_1^2}{R_2^2} \sin^2 \delta\right)}$$

or

$$r_{max} = \lambda \left[1 - \frac{\frac{Ve'}{E_0}}{\cos^2 \alpha \left(1 - \frac{R_1^2}{R_2^2} \sin^2 \delta\right)}\right]$$

The angle α_{max} is always potential-limited, as was θ in the plane case, since the component of fragment velocity directed towards the outer electrode is proportional to $\cos \alpha$; it approaches zero as α approaches $\pi/2$. However, α_{max} is a function of δ as well as V; the fragments are not emitted into an azimuthally symmetric field as they were in the plane and spherical cases, and a single angle such as θ is insufficient to prescribe their trajectories. Just as was done previously for θ_{max}, α_{max} can be found by setting $r_{max} = 0$, giving now the largest α at which fragments of energy E_0 can still penetrate the electric field for each value of δ:

$$\cos^2 \alpha_{max} = \frac{\frac{Ve'}{E_0}}{1 - \frac{R_1^2}{R_2^2} \sin^2 \delta}$$

or

$$\alpha_{max} = \cos^{-1}\left(\sqrt{\frac{\dfrac{Ve'}{E_0}}{1 - \dfrac{R_1^2}{R_2^2}\sin^2\delta}}\right)$$

The angle δ_{max} is similar to θ_{max} in the spherical case in that it is a function of V for large V, and is $\pi/2$ for small V, where the potential does not prevent emission downward into the fuel layer. The quantity δ reaches its largest value when none of the fragment energy is devoted to axial motion ($\alpha = 0$). Thus, for the potential-limited case:

$$\frac{\dfrac{Ve'}{E_0}}{1 - \dfrac{R_1^2}{R_2^2}\sin^2\delta_{max}} = 1$$

or

$$\sin^2\delta_{max} = \frac{R_2^2}{R_1^2}\left(1 - \frac{Ve'}{E_0}\right)$$

$$\delta_{max} = \sin^{-1}\left(\frac{R_2}{R_1}\sqrt{1 - \frac{Ve'}{E_0}}\right)$$

when

$$\frac{Ve'}{E_0} \geq \left(1 - \frac{R_1^2}{R_2^2}\right)$$

For the surface-limited case:

$$\delta_{max} = \frac{\pi}{2}$$

when

$$\frac{Ve'}{E_0} < \left(1 - \frac{R_1^2}{R_2^2}\right)$$

As before, there exist thick-layer and thin-layer cases:

$$\tau \geq \lambda \left(1 - \frac{Ve'}{E_0}\right) \qquad \text{thick layer}$$

$$\tau < \lambda \left(1 - \frac{Ve'}{E_0}\right) \qquad \text{thin layer}$$

The dividing surface between energy-limited and thickness-limited emitting volumes is now no longer a circular cone of angle θ. It is delineated by α_τ, which is a function of δ, and by δ_τ, which is the largest δ occurring on the dividing surface (at $\alpha_\tau = 0$). At the dividing surface:

$$r_{max} = \frac{\tau}{\cos \alpha_\tau \cos \delta} = \lambda \left[1 - \frac{\dfrac{Ve'}{E_0}}{\cos^2 \alpha_\tau \left(1 - \dfrac{R_1^2}{R_2^2} \sin^2 \delta\right)}\right]$$

or

$$\cos \alpha_\tau = \frac{\tau}{2\lambda \cos \delta} + \sqrt{\frac{\tau^2}{4\lambda^2 \cos^2 \delta} + \frac{\dfrac{Ve'}{E_0}}{\left(1 - \dfrac{R_1^2}{R_2^2} \sin^2 \delta\right)}}$$

When $\alpha_\tau = 0$:

$$\frac{\tau}{\lambda \cos \delta_\tau} = 1 - \frac{\dfrac{Ve'}{E_0}}{\left(1 - \dfrac{R_1^2}{R_2^2} \sin^2 \delta_\tau\right)}$$

This is a cubic equation in $\cos \delta_\tau$ or $\sin \delta_\tau$, from which δ_τ must be found.

The fragment current reaching the anode for the above four cylindrical cases is:

$$\eta = \frac{N}{\pi} \int_0^{\pi/2} d\delta \int_0^{\alpha_{max}} d\alpha \int_0^{r_{max}} dr \cos^2 \alpha \cos \delta \tag{7}$$

for

$$\tau \geq \lambda \left(1 - \frac{Ve'}{E_0}\right); \qquad \frac{Ve'}{E_0} < \left(1 - \frac{R_1^2}{R_2^2}\right)$$

$$\eta = \frac{N}{\pi} \int_0^{\delta_{max}} d\delta \int_0^{\alpha_{max}} d\alpha \int_0^{r_{max}} dr \cos^2 \alpha \cos \delta \tag{8}$$

for

$$\tau \geq \lambda \left(1 - \frac{Ve'}{E_0}\right); \qquad \frac{Ve'}{E_0} \geq \left(1 - \frac{R_1^2}{R_2^2}\right)$$

$$\eta = \frac{N}{\pi} \left(\int_0^{\delta_\tau} d\delta \int_0^{\alpha_\tau} d\alpha \int_0^{\tau/\cos\alpha \cos\delta} dr \cos^2 \alpha \cos \delta \right.$$

$$\left. + \int_0^{\delta_\tau} d\delta \int_{\alpha_\tau}^{\alpha_{max}} d\alpha \int_0^{r_{max}} dr \cos^2 \alpha \cos \delta + \int_{\delta_\tau}^{\pi/2} d\delta \int_0^{\alpha_{max}} d\alpha \int_0^{r_{max}} dr \cos^2 \alpha \cos \delta \right) \tag{9}$$

for

$$\tau < \lambda \left(1 - \frac{Ve'}{E_0}\right); \qquad \frac{Ve'}{E_0} < \left(1 - \frac{R_1^2}{R_2^2}\right)$$

$$\eta = \frac{N}{\pi} \left(\int_0^{\delta_\tau} d\delta \int_0^{\alpha_\tau} d\alpha \int_0^{\tau/\cos\alpha \cos\delta} dr \cos^2 \alpha \cos \delta \right.$$

$$\left. + \int_0^{\delta_\tau} d\delta \int_{\alpha_\tau}^{\alpha_{max}} d\alpha \int_0^{r_{max}} dr \cos^2 \alpha \cos \delta + \int_{\delta_\tau}^{\delta_{max}} d\delta \int_0^{\alpha_{max}} d\alpha \int_0^{r_{max}} dr \cos^2 \alpha \cos \delta \right) \tag{10}$$

for

$$\tau < \lambda \left(1 - \frac{Ve'}{E_0}\right) \; ; \qquad \frac{Ve'}{E_0} \geq \left(1 - \frac{R_1^2}{R_2^2}\right)$$

Where

$$\delta_{max} = \sin^{-1}\left(\frac{R_2}{R_1}\sqrt{1 - \frac{Ve'}{E_0}}\right)$$

$$\alpha_{max} = \cos^{-1}\left(\sqrt{\frac{\dfrac{Ve'}{E_0}}{1 - \dfrac{R_1^2}{R_2^2}\sin^2\delta}}\right)$$

$$\tau_{max} = \lambda\left[1 - \frac{\dfrac{Ve'}{E_0}}{\cos^2\alpha\left(1 - \dfrac{R_1^2}{R_2^2}\sin^2\delta\right)}\right]$$

$$\alpha_{\tau} = \cos^{-1}\left(\frac{\tau}{2\lambda\cos\delta} + \sqrt{\frac{\tau^2}{4\lambda^2\cos^2\delta} + \frac{\dfrac{Ve'}{E_0}}{\left[1 - \dfrac{R_1^2}{R_2^2}\sin^2\delta\right]}}\right)$$

δ_{τ} is found from:

$$\frac{\tau}{\lambda\cos\delta_{\tau}} = 1 - \frac{\dfrac{Ve'}{E_0}}{\left(1 - \dfrac{R_1^2}{R_2^2}\sin^2\delta_{\tau}\right)}$$

IV. NUMERICAL SOLUTION

While the η can be obtained in analytic form for plane and spherical geometries, it is necessary to utilize numerical computation to obtain the integrals for cylindrical cases and also to determine θ_τ and δ_τ. In actual practice, it was simpler to perform all the integrations numerically, utilizing the IBM 704 computer and a single program which carried out the following operations:

Integral I

$$\mathcal{E} = \frac{E_0}{E'} \frac{V}{2\tau} \int_0^{\theta_{max}} d\theta \int_0^{r_{max}} dr \sin\theta \cos\theta$$

where

$$r_{max} \text{ is the smaller of } \begin{cases} 1 - \dfrac{V}{\cos^2\theta} \\ \dfrac{\tau}{\cos\theta} \end{cases}$$

$$\cos^2\theta_{max} = V$$

Integral II

$$\mathcal{E} = \frac{E_0}{E'} \frac{V}{2\tau} \int_0^{\theta_{max}} d\theta \int_0^{r_{max}} dr \sin\theta \cos\theta$$

where

$$r_{max} \text{ is the smaller of } \begin{cases} \left[1 - \dfrac{V}{1 - \dfrac{R_1^2}{R_2^2} \sin^2\theta} \right] \\ \dfrac{\tau}{\cos\theta} \end{cases}$$

$$\sin^2 \theta_{max} \text{ is the smaller of } \begin{cases} \dfrac{R_2^2}{R_1^2}(1 - V) \\ 1 \end{cases}$$

Integral III

$$\mathcal{E} = \frac{E_0}{E'} \frac{V}{\pi \tau} \int_0^{\delta_{max}} d\delta \int_0^{\alpha_{max}} d\alpha \int_0^{r_{max}} dr \cos^2 \alpha \cos \delta$$

where

$$r_{max} \text{ is the smaller of } \begin{cases} \left[1 - \dfrac{V}{\cos^2 \alpha \left(1 - \dfrac{R_1^2}{R_2^2} \sin^2 \alpha\right)}\right] \\ \dfrac{\tau}{\cos \alpha \cos \delta} \end{cases}$$

$$\cos^2 \alpha_{max} = \frac{V}{1 - \dfrac{R_1^2}{R_2^2} \sin^2 \delta}$$

$$\sin^2 \delta_{max} \text{ is the smaller of } \begin{cases} \dfrac{R_2^2}{R_1^2}(1 - V) \\ 1 \end{cases}$$

These integrals are just the efficiencies in the plane, spherical, and cylindrical cases discussed above expressed in the units

V in units of E_0/e'

τ in units of λ

An energy ratio $E_0/E' = 0.9$ was assumed.

V. RESULTS

The quantity $\mathcal{E}$ has been calculated for the three geometries over the potential range $V = 0$ to $V = E_0/e'$, for fuel layer thicknesses from 0 to 10λ. In spherical and cylindrical geometries, a series of R_2/R_1 values has been utilized which appears to cover the most useful range for reactors in space application. Results are shown in Fig. 2 through 4.

VI. ANALYTICAL SOLUTIONS

For purposes of completeness, the solutions which have been obtained in analytical form can be summarized:

Plane Parallel Electrodes:

$$\mathcal{E} = \frac{Ve'\lambda}{4E'\tau}\left[1 - \frac{Ve'}{E_0} + \frac{Ve'}{E_0}\ln\left(\frac{Ve'}{E_0}\right)\right] \qquad (11)$$

for

$$\tau \geq \lambda\left(1 - \frac{Ve'}{E_0}\right)$$

$$\mathcal{E} = \frac{Ve'}{2E'}\left[1 - \frac{\tau}{4\lambda} - \frac{1}{4}\sqrt{\frac{\tau^2}{\lambda^2} + 4\frac{Ve'}{E_0}} - \frac{\lambda Ve'}{\tau E_0}\ln\left(\frac{\frac{\tau}{\lambda} + \sqrt{\frac{\tau^2}{\lambda^2} + 4\frac{Ve'}{E_0}}}{\sqrt{4\frac{Ve'}{E_0}}}\right)\right] \qquad (12)$$

for

$$\tau < \lambda\left(1 - \frac{Ve'}{E_0}\right)$$

Spherical Electrodes:

$$\mathscr{E} = \frac{Ve'\lambda}{4E'\tau}\left[1 + \frac{Ve'}{E_0}\,\frac{R_2^2}{R_1^2}\,\ln\left(1 - \frac{R_1^2}{R_2^2}\right)\right] \tag{13}$$

for

$$\tau \geq \lambda\left(1 - \frac{Ve'}{E_0}\right); \qquad \frac{Ve'}{E_0} < \left(1 - \frac{R_1^2}{R_2^2}\right)$$

$$\mathscr{E} = \frac{Ve'\lambda}{4E'\tau}\,\frac{R_2^2}{R_1^2}\left[1 - \frac{Ve'}{E_0} + \frac{Ve'}{E_0}\,\ln\left(\frac{Ve'}{E_0}\right)\right] \tag{14}$$

for

$$\tau \geq \lambda\left(1 - \frac{Ve'}{E_0}\right); \qquad \frac{Ve'}{E_0} \geq \left(1 - \frac{R_1^2}{R_2^2}\right)$$

$$\mathscr{E} = \frac{Ve'}{2E'\tau}\left\{\tau\left[1 - \cos\theta_\tau\right] + \frac{\lambda}{2}\left[1 + \frac{Ve'}{E_0}\,\frac{R_2^2}{R_1^2}\,\ln\left(1 - \frac{R_1^2}{R_2^2}\right)\right.\right.$$

$$\left.\left. - \sin^2\theta_\tau - \frac{Ve'}{E_0}\,\frac{R_2^2}{R_1^2}\,\ln\left(1 - \frac{R_1^2}{R_2^2}\sin^2\theta_\tau\right)\right]\right\} \tag{15}$$

for

$$\tau < \lambda\left(1 - \frac{Ve'}{E_0}\right); \qquad \frac{Ve'}{E_0} < \left(1 - \frac{R_1^2}{R_2^2}\right)$$

$$\mathscr{E} = \frac{Ve'}{2E'\tau}\left\{\tau\left[1 - \cos\theta_\tau\right] + \frac{\lambda}{2}\left[\frac{R_2^2}{R_1^2}\left(1 - \frac{Ve'}{E_0}\right) + \frac{Ve'}{E_0}\,\frac{R_2^2}{R_1^2}\,\ln\left(\frac{Ve'}{E_0}\right)\right.\right.$$

$$\left.\left. - \sin^2\theta_\tau - \frac{Ve'}{E_0}\,\frac{R_2^2}{R_1^2}\,\ln\left(1 - \frac{R_1^2}{R_2^2}\sin^2\theta_\tau\right)\right]\right\} \tag{16}$$

for

$$\tau < \lambda \left(1 - \frac{Ve'}{E_0}\right) ; \qquad \frac{Ve'}{E_0} \geq \left(1 - \frac{R_1^2}{R_2^2}\right)$$

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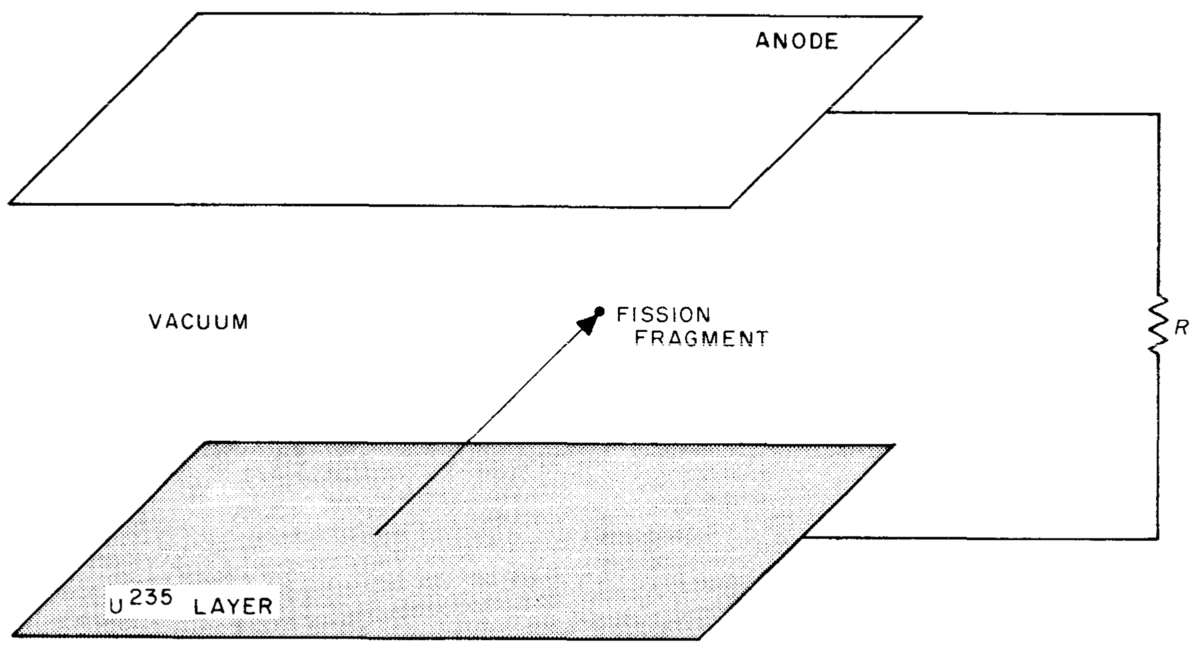

Fig. 1. Simplified fission cell

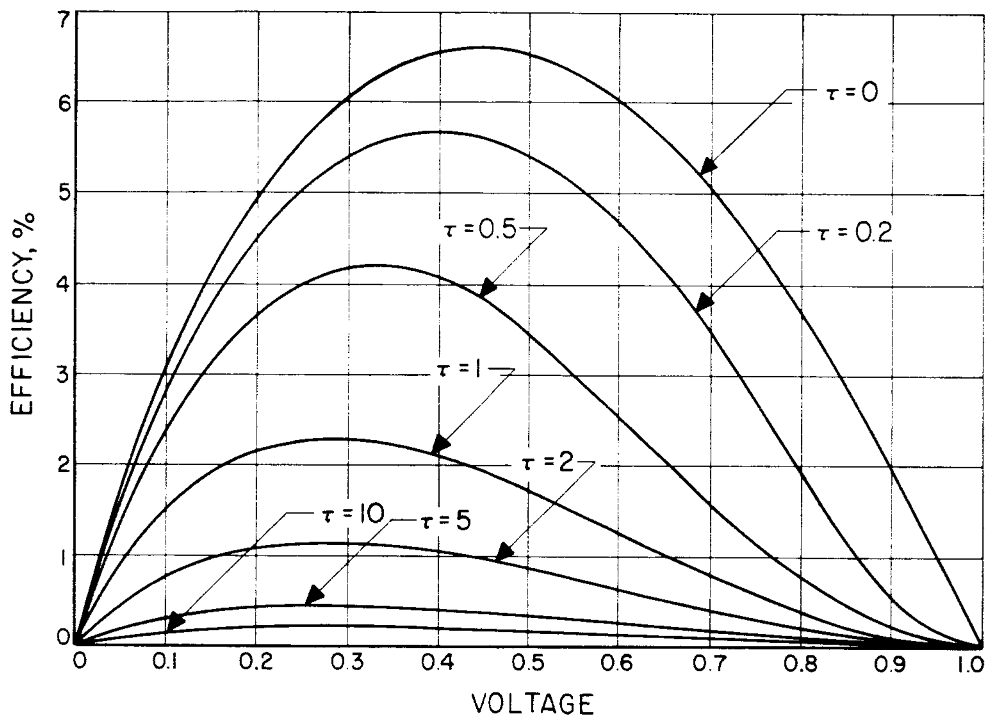

Fig. 2. Efficiency vs voltage for parallel plane electrode fission cell

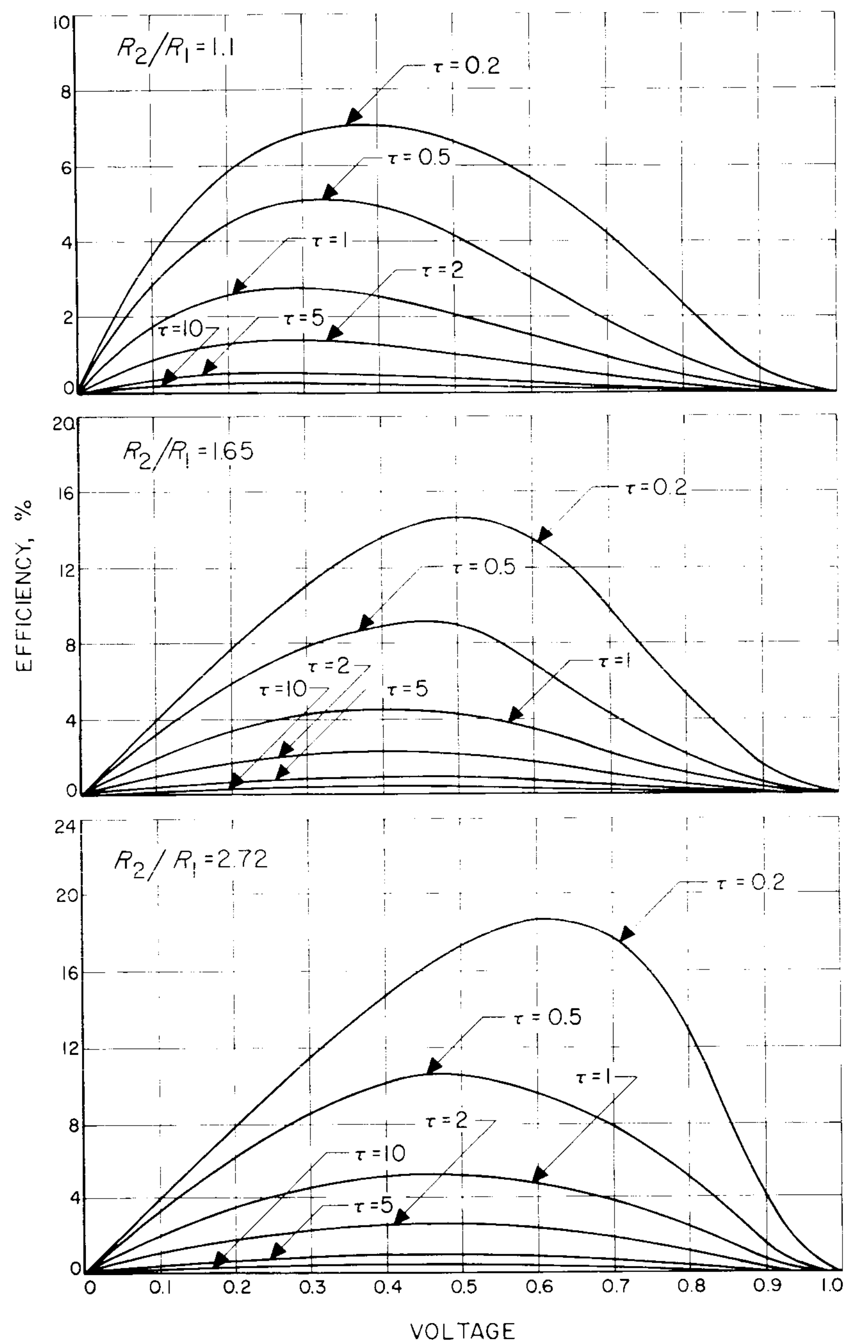

Fig. 3. Efficiency vs voltage for spherical electrode fission cell

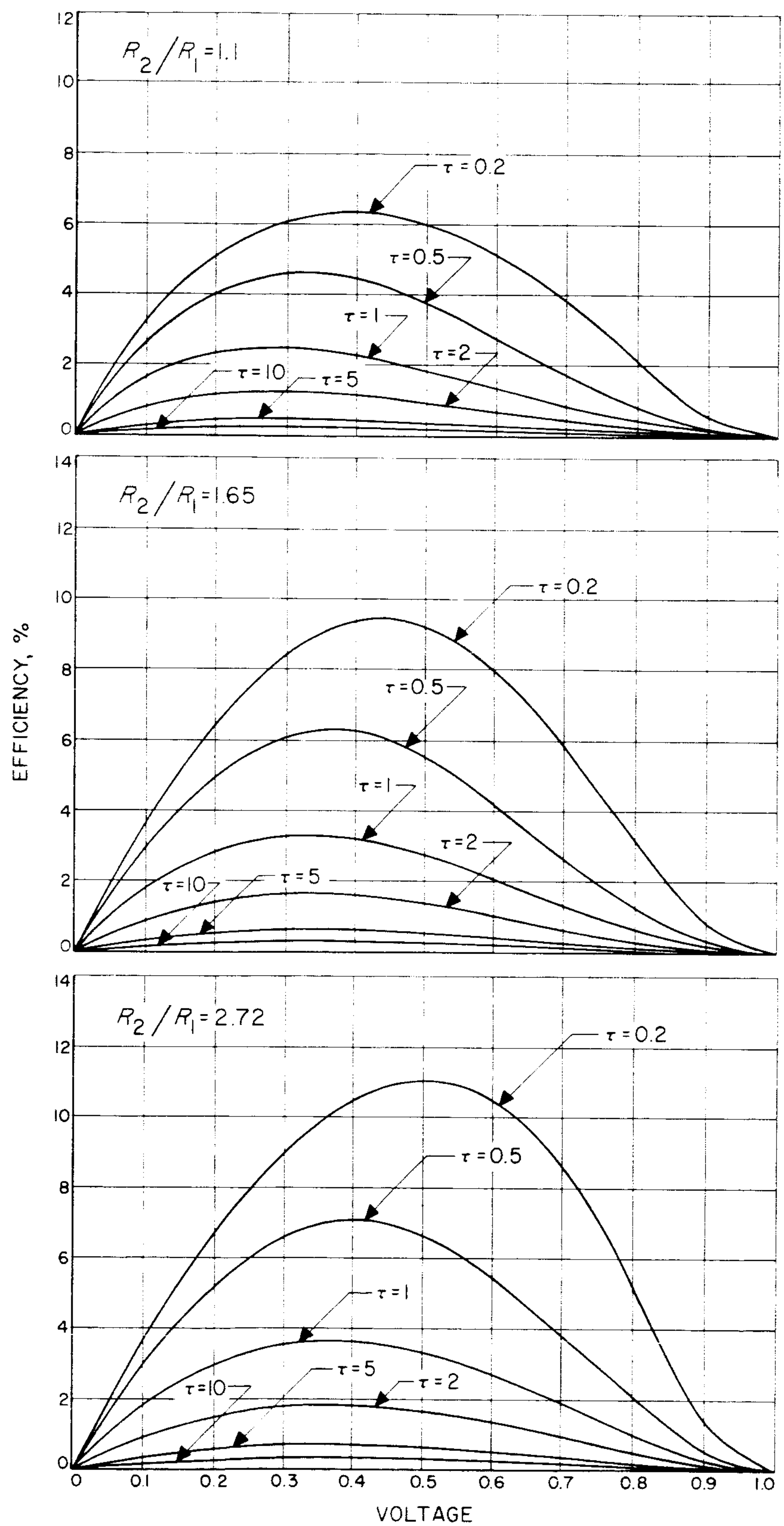

Fig. 4. Efficiency vs voltage for cylindrical electrode fission cell